



Preface

This book has been germinating for a long time. John Conway has always been interested in geometrical groups, for many of which he devised particular notations when he was teaching at Cambridge University. However, after he moved to Princeton University in 1985 and Bill Thurston told him of the orbifold idea, he dropped those notations forever and devised the signature notation used in this book. He then became Thurston's most avid prophet, lecturing on the theory to scores of audiences—ranging from the Princeton Rug Society to the International Congress of Mathematicians!

One of those audiences contained the young graduate student Heidi Burgiel, who was taking notes on the talk for distribution during the conference. Heidi went on to complete a graduate program in combinatorics and discrete geometry. Years later, when John spent some time at Northwestern University, Heidi offered to “write something else up” with John, but in the end they decided to write the same theory in more detail as a proposed book. That book has been growing ever since.

All they had intended to write was the content of what is now Part I—an elementary introduction to the orbifold signature notation. But then came the idea of writing a second part that would extend the signature to color symmetry. At this point it became clear that Chaim Goodman-Strauss would make an excellent addition to the team of authors. Chaim had been preaching the gospel of the orbifold signature on his own and was known for his gorgeous illustrations.

More topics burst into bloom at various seasons. When Conway, Delgado, Huson, and Thurston used the signature to re-enumerate the three-dimensional space groups, it seemed a good idea to incorporate this also in the second part. That “second part” is now Parts II and III.

Much of the book was written in hectic three-day sessions on the few occasions when all three of us could get together—this paragraph is being finalized on the way to the Tampa airport, days before the book is sent to press. We usually managed to write several chapters in each session, often including one that only arose just then. For example, at one session, Chaim said “we could perhaps do Heesch types,” and an hour later Chapter 15 was complete. Just after completing the next section of this introduction (which describes what’s new to this book), the three of us celebrated at a restaurant, discussed “Archimedean tilings,” and Chaim and John discovered the “Archifold notation” that characterizes such things as they walked home after the meal. The next day this too was in the book. Of course, it often took Chaim years to catch up with the illustrations.

What’s New in This Book?

Many of the results and proofs in this book are new, or nearly new, in the sense that their only previous appearances have been in the scholarly papers (often involving one of us) that are cited in the appropriate chapters. These new things are

- the orbifold signature,
- the statement of our Magic Theorem,
- its use to enumerate symmetry types
(however, we should point out that a few decades ago, MacBeath introduced his own signature that is in fact equivalent to ours—but more complicated—and used it in the same way),
- Conway’s “zip proof” of the classification of surfaces,
- uniform presentations for all the groups,
- their proof,
- our analysis and notation for color symmetry,
- the p -color types for all primes p ,
- the simplified enumeration of Heesch types,

- the Besche-Eick-OBrien table of group numbers,
- the extension of all of the above geometrical theory to hyperbolic groups,
- a new proof of the abstract distinctness of infinite groups with compact orbifolds,
- the explanation of isospectral “drums” via hyperbolic groups,
- the classification of Archimedean tilings in the hyperbolic plane,
- generalized Schläfli symbols,
- Architectonic and Catoptric 3-tessellations,
- the new space group notations and a panoply of objects with prime space symmetries,
- names and enumeration of platycosms,
- a list of Archimedean 4-polytopes.

Even when the results are old, our exposition is new.

We are also proud of our exposition and illustrations. Chaim Goodman-Strauss assures our readers that his software and illustrations are available for sale and licensing.

We are relieved that now the book is in print, bringing the orbifold signature to the world. This would not have happened without help from many people including Robert Strauss, Troy Gilbert, Marc Culler, Tom Moore, Charlotte Henderson, Alice and Klaus Peters, Bill Thurston, Silvio Levy, Peter Doyle, Natasha Jonaska, Daniel Huson, Olaf Delgado Friedrichs, Doris Schattschneider, Marjorie Senechal, Javier Bracho, our students, and our colleagues; and the patience and sympathy of our partners Diana, Kendall, and Rachel. We thank the institutions that supported our work, including Princeton University, the University of Arkansas, the Universidad Nacional Autónoma de México, Northwestern University, the University of Illinois at Chicago, Bridgewater State College, and the National Science Foundation.