Introduction to Part I

Symmetries and symmetric patterns surround us throughout our lives. The aim of the first part of this book is to describe and enumerate all the symmetries found in repeating patterns on surfaces. To prove that our enumeration is accurate, we then explain the beautiful ideas from topology and algebra that form the basis for our conclusions.

We start with a problem—enumerating symmetric patterns. We then introduce tools for solving this problem and complete the enumeration. But then we are presented with a second problem—demonstrating that these tools work the way we claim, that there is a solid mathematical foundation beneath our results. Again, we solve this problem with some tools, then present the mathematics supporting the use of those tools. In this way, each chapter reduces the problems left by the preceding chapter to another problem whose solution is postponed to the following chapter.

This is a departure from the traditional practice of building a theory starting with basic principles and working toward the ultimate goal of proving some final theorem. We believe that our backward approach will be successful because it allows us to present one concept at a time, at the cost of always postponing the proof of just one thing to the next chapter. We hope also that the argument will be clearer when presented in a single logical thread, of the form $A \Leftarrow B \Leftarrow C \Leftarrow ... \Leftarrow Z$.

The first chapter is a gentle introduction to symmetry. Chapter 2 introduces the four fundamental features that we use to classify symmetry. In Chapter 3 we state our Magic Theorem and apply it to find the 17 possible types of repeating planar patterns, while Chapters 4 and 5 perform a similar service for spherical and frieze patterns, respectively. The Magic Theorem is deduced in Chapter 6 from Euler's Theorem, which is itself proved in Chapter 7. Finally, Chapter 8 gives our new proof of the classification of surfaces, and Chapter 9 illustrates the orbifolds that underlie our theory.