

Errata for Differential Geometry of Curves and Surfaces

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The following list reflects errata found in the text as of August 10, 2011.

Chapter 1

Page 3 line 10: $t \in [0, 2\pi]$ should be $t \in [0, 2\pi/\omega]$.

Page 9 line -1: The distance formula in the display equation should be

$$\sqrt{\|\vec{p} - \vec{b}\|^2 - \frac{(\vec{a} \cdot (\vec{p} - \vec{b}))^2}{\|\vec{a}\|^2}} = \frac{\|\vec{a} \times (\vec{p} - \vec{b})\|}{\|\vec{a}\|},$$

Page 29 line 23: Definition 1.4.4, in the last line “of order 2.” should be “of order at least 2.”

Page 36 line 1: “Given a function $\kappa_g(s)$,” should be “Given a piecewise continuous function $\kappa_g(s)$,”. (This is necessary to ensure that $\kappa_g(s)$ is integrable and that $\cos \theta(s)$ and $\sin \theta(s)$ are integrable.)

Page 38 line 7: In Problem 1.5.2, $\vec{x}'(s) = (1, 0)$ should be the initial condition $\vec{x}'(0) = (1, 0)$.

Chapter 2

Page 41 line 9: In the assumptions on \mathcal{C} we must also assume that \mathcal{C} is positively oriented.

Page 53 line 21: In Section 2.4, we assume that all curves can be parametrized with a parametrization of class C^2 .

Chapter 3

Page 70 line 8: “parametrized curve” should be “parametrized, regular curve.”

Page 80 line 3-5: The interpretation of the sign of torsion is reversed. We have $\tau(t_0) > 0$ at $\vec{x}(t_0)$ when the curve comes up through the osculating plane and we have $\tau(t_0) < 0$ at $\vec{x}(t_0)$ when the curve goes down through the osculating plane. (Figure 3.5 is correct.)

Page 81 line -1: “contact of order 3” should be “contact of order at least 3.”

Chapter 4

Page 88 line 10: In Stokes’s Theorem, we must also assume that \mathcal{C} is oriented according to the right hand rule with respect to the oriented surface \mathcal{S} .

Page 91 line -4: $AB = 2 \sin\left(\frac{AB}{2}\right)$ should be $AB = 2 \sin\left(\frac{\overline{AB}}{2}\right)$.

Page 91 line -3: “Let Γ be a regular curve” should be “Let Γ be a closed regular curve”.

Page 92 line 4: “ $L \leq 2\pi$ ” should be “ $L < 2\pi$ ”.

Page 97 line 12: “ $\vec{X}(t)$ ” should be “ $\vec{x}(t)$ ”.

Page 102 line -2: “in Figure 4.6” should be “in the oriented diagram on the right in Figure 4.6”

Chapter 5

Page 114 line 13: “the x -axis” should be “the y -axis.”

Page 120 line 7: The indices m and n in the matrix of partial derivatives are reversed.

Page 121 line 5: “ dF_q does not have maximal rank” should be “ dF_q does not exist or does not have maximal rank”.

Page 121 line -8: “ $U \in \mathbb{R}^2$ ” should be “ $U \subseteq \mathbb{R}^2$ ”.

Page 123 line -6: “ $|x \geq 0$ if $y = 0$ ” should be “ $|x \geq 0$ and $y = 0$ ”.

Page 124 line 3: “ $|x \leq 0$ if $z = 0$ ” should be “ $|x \leq 0$ and $z = 0$ ”.

Page 131 line 6: $F(U) = U'$ should be $F(U') = U$.

Page 133 line 2: dF_q should be $dF_{q'}$.

Page 137 line -2: “for any fixed $u_0 \in I$, along any curve $\vec{X}(u_0, v)$,” should read “for any fixed $t_0 \in I$, along any curve $\vec{X}(t_0, u)$,”

Page 138 line 18: “is a regular curve” should be “is a disjoint union of regular curves”

Chapter 6

Page 160 line 10: “two-dimensional subspace of \mathbb{R}^2 ” should be “two-dimensional subspace of \mathbb{R}^3 ”

Page 169 line 3: $2L_{12}st$ should be $2L_{12}(q)st$.

Page 169 line 10: $+\vec{X}_{vv}(u_0, v_0)(v - v_0)^2$ should be $+\frac{1}{2}\vec{X}_{vv}(u_0, v_0)(v - v_0)^2$.

Page 174 line -3: The display equation should read:

$$-L_{ij} = \vec{N}_i \cdot \vec{X}_j = \left(\sum_{k=1}^2 a_i^k \vec{X}_k \right) \cdot \vec{X}_j,$$

Page 175 line 2,7: The author incorrectly listed the transpose of the matrix of the differential of the Gauss map instead of the matrix itself. Equation (6.25) should read

$$-\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix}.$$

Consequently, Equation (6.26), which defines the Weingarten equations should read

$$\begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix} = - \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}.$$

Page 179 line 14,15: Lg^{-1} in both of those lines should be $g^{-1}L$.

Page 180 line -1: Because of the previous comment, the matrix (a_i^j) is the transpose of what is given in the text.

Page 181 line -5: Equation (6.30) should be

$$\begin{pmatrix} v' \\ -u' \end{pmatrix} \cdot dn_p \begin{pmatrix} u' \\ v' \end{pmatrix}.$$

Page 181 line -4: “Any curve” should be “Any regular curve”

Page 182 line -7: The display equation should be

$$L_{11}g_{12} = L_{12}g_{11} \quad L_{11}g_{22} = L_{22}g_{11} \quad L_{22}g_{12} = L_{12}g_{22}.$$

Page 183 line 8: $-Lg^{-1}$ should be $-g^{-1}L$.

Page 183 line -3: “ $L_{21}g_{11} = 0$ or $L_{12}g_{22} = 0$ ” should be “ $L_{21}g_{11} = 0$ and $L_{12}g_{22} = 0$ ”.

Page 186 line 22: Delete the $\vec{T}(t)$ in the last parentheses.

Page 187 line 2: The display equation should read:

$$L_{11} = \frac{-fh'}{\sqrt{f'(v)^2 + h'(v)^2}}, \quad L_{12} = L_{21} = 0, \quad L_{22} = \frac{f''h' - f'h''}{\sqrt{f'(v)^2 + h'(v)^2}}.$$

Page 187 line 6: The display equation should read:

$$\kappa_1(u, v) = -\frac{h'(v)}{f(v)\sqrt{f'(v)^2 + h'(v)^2}}, \quad \kappa_2(u, v) = \frac{f''(v)h'(v) - f'(v)h''(v)}{((f'(v))^2 + (h'(v))^2)^{3/2}},$$

Page 189 line 6: “regular surface” should be “regular oriented surface”

Page 189 line -9: “mean curvature of S ” should be “mean curvature (up to sign) of S ”

Page 191 line 10: $-Lg^{-1}$ should be $-g^{-1}L$.

Page 192 line 19: The L_{ij} matrix is off by a sign.

Page 198 line 14: “a curve C ” should be “a simple, closed curve C ”

Page 201 line 9: $\vec{\beta}' \times \vec{w} = (\vec{\beta}' \times \vec{w}) \cdot \vec{w}' / \|\vec{w}'\|$ should be $\|\vec{\beta}' \times \vec{w}\| = |(\vec{\beta}' \times \vec{w}) \cdot \vec{w}'| / \|\vec{w}'\|$

Page 201 line 18: Equation (6.44) should read

$$(g_{ij}) = \begin{pmatrix} \|\vec{\beta}'\|^2 + u^2\|\vec{w}'\|^2 & \vec{\beta}' \cdot \vec{w}' \\ \vec{\beta}' \cdot \vec{w}' & 1 \end{pmatrix}$$

Page 202 line 20-21: “is a cone if and only if $\vec{\alpha}'(t) = 0$ ” should be “is a cone if $\vec{\alpha}'(t) = 0$ ”

Page 204 line 11: $D' \in U$ should be $D' \subseteq U$.

Page 205 line 8: $\|\vec{X}_u \times \vec{X}_v\| \neq 0$ should be $\|\vec{X}_u \times \vec{X}_v\| \neq 0$

Page 207 line 10: In Problem 6.6.10, the beginning “Show that” should read “Prove or disprove that”.

Page 207 line 14-15: “parametrized by $\vec{Z}_t(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(u, v)$ ” should be “parametrized by $\vec{Z}^t(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(u, v)$ ”

Chapter 7

Page 216 line -3: Remove the comma in the display equation.

Page 222 line -3: The display equation should read

$$a_j^i = -g^{ik}L_{kj}.$$

Page 244 line -9: The display equation for the parametrization of the torus should be

$$\vec{X}(u, v) = ((b + a \cos v) \cos u, (b + a \cos v) \sin u, a \sin v),$$

Page 244 line -2,-1: “Problem 6.5.5” should be “Example 6.5.5.”

Page 255 line -4: “18 partial differential equation” should be “18 partial differential equations”

Chapter 8

Page 281 line 6: “direction of \vec{v} .” should “direction of \vec{u} .”

Page 281 line 14: $F : (R)^n \rightarrow \mathbb{R}^n$ should be $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$.

Page 298 line -6: In the definition of piecewise regular, one assumes that $\vec{\alpha}'(t)$ has nonzero one-sided limits at each t_i but such that the one-sided limits are distinct.

Page 299 line 5: “collinear” should be “collinear and pointing in opposite directions”.

Page 300 line 2: “orange” should be “green”

Page 300 line 16: Example 8.1.9 should be Example 8.4.4

Page 303 line 18: “that it consider a” should be “that it considers a”.

Page 310 line 4: “independedent” should be “independent”

Page 314 line 5: The display equation should read

$$\iint_S K dS > 0.$$

Page 314 line -6: “propositions through from 23 definitions” should be “propositions from 23 definitions”

page 315 line -1: “piecewise regular, simple, closed curve.” should be “piecewise regular, simple, positively oriented, closed curve.”